

Lack of observational evidence for quantum structure of space–time at Planck scales ¹

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ABSTRACT

It has been noted (Lieu & Hillmann, 2002) that the cumulative affect of Planck–scale phenomenology, or the structure of space–time at extremely small scales, can be lead to the loss of phase of radiation emitted at large distances from the observer. We elaborate on such an approach and demonstrate that such an effect would lead to an apparent blurring of distant point–sources. Evidence of the diffraction pattern from the HST observations of SN 1994D and the unresolved appearance of a Hubble Deep Field galaxy at $z=5.34$ lead us to put stringent limits on the effects of Planck–scale phenomenology.

Subject headings: gravitation–time

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1. Introduction

It is generally believed that a description of the gravity consistent with quantum theory (quantum gravity) should imply properties of the space–time much different from the conventional ones, when the latter is being observed at the so–called *Planck scale*. Such a Planck scale is obtained as a combination of fundamental constants and corresponds to a characteristic length $l_P \approx 1.6 \times 10^{-35}$ m and time interval $t_P \approx 5.4 \times 10^{-44}$ s given by:

$$l_P = ct_P = \sqrt{\frac{G\hbar}{c^3}} \quad (1)$$

Space and time, when observed at such scales, are expected to exhibit a grainy, fuzzy or a foam-like structure, as depicted by several authors (see for instance Rovelli 1998, Garay 1998, Kempf 1999). The operational definition of measurement of a length or of a time–interval should be affected by such property of space–time (Wigner, 1957; Salecker & Wigner, 1958; Adler & Santiago, 1999) and one can conceive several *gedanken* experiments that should be affected by the so–called Planck Scale Phenomenology (hereafter PSP, see Amelino–Camelia, 2001a).

In spite of the extremely small size of l_P , recently several authors pointed out that the *systematic accumulation* of such effect during the long journey of a photon propagating through a space–time affected by PSP could lead to observable consequences. Several possible measurements has been proposed so far (Amelino–Camelia et al., 1997, 1998; Ellis, Mavromatos & Nanopoulos, 2000; Ng & van Dam, 1999) and eventually later criticized (Adler et al., 2000).

Most recently Lieu & Hillman, 2002 (hereafter LH02) suggested that differential phase measurements of light propagated over a long distance, as implicitly made by interferometry of an extragalactic source, can place much tighter constraints on PSP. They derived the effect on the random phase variation as depending upon the ratio of the photon wavelength λ and the Planck length l_P .

It is important to point out that the effects described by LH02 refers to a model of PSP leading to random variations of the light phase, while others exhibit a definite modification of radiation behavior for a given wavelength (Jacobson, Liberati & Mattingly, 2002; Amelino–Camelia 2002). In other words any spacetime structure model that yields a definite modification at a given wavelength is unconstrained by the random phase approach.

Furthermore, we are aware that LH02 has been the subject of even more recent criticism (Ng, van Dam & Christiansen, 2003) essentially based upon the idea that such random

perturbation of space–time should add incoherently along the propagation path, leading to a square-root dependency upon the distance between the source and the observer. We just here note that such assumption is at the basis of other PSPs (Amelino Camelia, 1994) already ruled out by experimental verifications (Ng & van Dam, 1999) and that other theories does not incorporate such dependencies too (Károlyhazy, 1966; Ng & van Dam, 1994).

In this letter we argue that the use of diffraction, as an interferometry effect by a telescope dish, can put stringent limits on the PSP with random phase variations.

2. Single aperture observations

Following LH02 we assume that the error in the phase of a wavefront is just a different way of expressing the impossibility of measuring, by means of light at wavelength λ , a distance L with a precision ΔL such that:

$$\frac{\Delta L}{L} < a_0 \left(\frac{l_P}{\lambda} \right)^\alpha \quad (2)$$

where the parameters a_0 and α characterize the theory being tested. For instance $\alpha = 1/2$ corresponds to the random–walk approach (Amelino–Camelia, 2000) while $\alpha = 2/3$ corresponds to the holography principle (Wheeler, 1982; Hawking, 1975) and $\alpha = 1$ is the natural choice in a linearized theory. The coefficient a_0 can be reasonably expected to be of the order of the unity but, according to Amelino–Camelia (2001b) it can be *a few orders of magnitude* below unity. This gives an idea of the region of the parameter space described by a_0 and α where a meaningful search should be done.

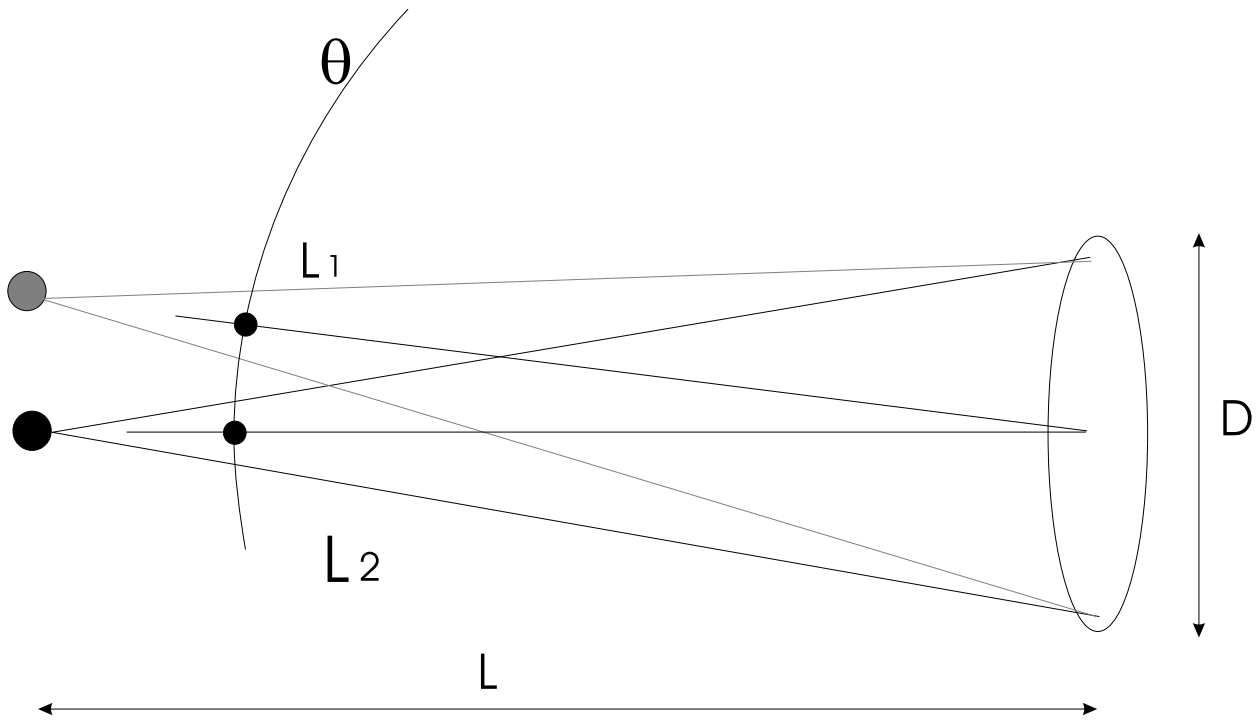


Fig. 1.— The observation of a light source at a distance L from the center of the telescope aperture. The distances between the source and two extremity positions on the aperture are denoted by L_1 and L_2 . A variation in L_1 and L_2 will result in an apparent displacement $\Delta\theta$ in the location of the source .

Let us assume, in fact, that the measurement of *any* distance between a light source and a telescope of diameter D is affected by the PSP described above, and translates into independent modification of the wavefront phase as determined from two distinct positions (this assumption, in the LH02 framework, corresponds to the, obviously verified, requirement of $D \gg l_P$). Let us consider the distances L_1 and L_2 as measured from a point source placed at a distance $L \approx L_1 \approx L_2$ from the two sides of a telescope of aperture D , see Fig.1. Any intrinsic variation ΔL in the wavefront along the two lines of sight will translate into an apparent angular shift $\Delta\theta$ given by:

$$\Delta\theta \approx \frac{\Delta L}{D} \quad (3)$$

where we did not co-add the (independent) uncertainties over the possible set of sightliness starting from any point of the telescope pupil (for instance, considering only L_1 and L_2 a $\sqrt{2}$ factor should be inserted into Eq.3) as this would not change the order of magnitude of the result.

It is important to emphasize that this result does not presume our knowledge of either L_1 or L_2 with the accuracy stated in Eq.(2). Actually the distance of any astronomical object is known with a much poorer accuracy than that required to test Eq.(2). The key point here is the *independence* of the accuracies on the measurements. Specifically, the difference in the optical paths joining various points of the telescope pupil and the (unresolved) source is randomly modified by PSP.

If the PSP effects of Eq. (2) are present, will this lead to a deterioration of any interference pattern (e.g. the Airy rings of a filled aperture) seen at the diffraction limit? Such a consequence is inevitable - to avoid it one must invoke the highly unlikely scenario of *correlated* fluctuations in the optical paths over all points along the entire span of the light footprint, which in general has a size $\gg l_P$ (excepting only an initial segment of paths, $\sim l_P \times L/D$ in length, which for the purpose of this work is an irrelevantly short (i.e. $\ll \lambda$) segment).

Thus it is reasonable to deduce that PSP leads to an apparent angular broadening of a light source placed at a distance L , as seen from a telescope of diameter D , given by:

$$\Delta\theta = a_0 \frac{L}{D} \left(\frac{l_P}{\lambda} \right)^\alpha \quad (4)$$

We compare such an angular broadening with the diffraction limit imposed by the telescope aperture by introducing a ratio η defined as:

$$\eta = \frac{\Delta\theta}{\lambda/D} = a_0 \frac{L}{\lambda} \left(\frac{l_P}{\lambda} \right)^\alpha \quad (5)$$

The meaning of η is that it directly influences fringe visibility in the case of an interferometer, or the Strehl ratio S of deterioration in point spread function in the case of a telescope. This is because one can write, following Sandler et al. 1994 and assuming that the broadening is equivalent to a blurring effect due to (e.g.) atmospheric disturbance, the following equation for S :

$$S = \exp(-\eta^2) \quad (6)$$

It is reasonable to adopt $\eta = 1$ as rough criterion for any experimental setup of this kind to secure a reliable test of PSP effects. At $\lambda = 1\mu\text{m}$, a representative wavelength for diffraction limited optical telescopes (including the $D = 2.4\text{m}$ aperture of the HST and $D = 8 \dots 10\text{m}$ class ground-based telescopes equipped with Adaptive Optics facilities), this criterion requires the observation of sources at a minimum distance $L_{\min} \approx 6.2 \times 10^{22} \text{ m} \approx 2.1 \text{ Mpc}$, as already noted in LH02, to detect or to rule out the case for $\alpha = 1$, $a_0 = 1$.

3. Astronomical benchmarks

A celestial object that appears extended can either be because it is genuinely so, or PSP causes a blurring of the image in the manner described above. To avoid confusion between the two possibilities, the best target choice is a Supernovae (SNe). This is because for a distant SNe, its angular size must remain considerably smaller than the telescope diffraction limit even if one assumes that the SNe shell has been expanding steadily at the speed of light since the initial explosion. Evidently, then, our purpose of scrutinizing PSP will be fulfilled by an investigation of an HST archival image of SN1994D, located at $L \approx 13.7\text{Mpc}$ (Patat et al., 1996).

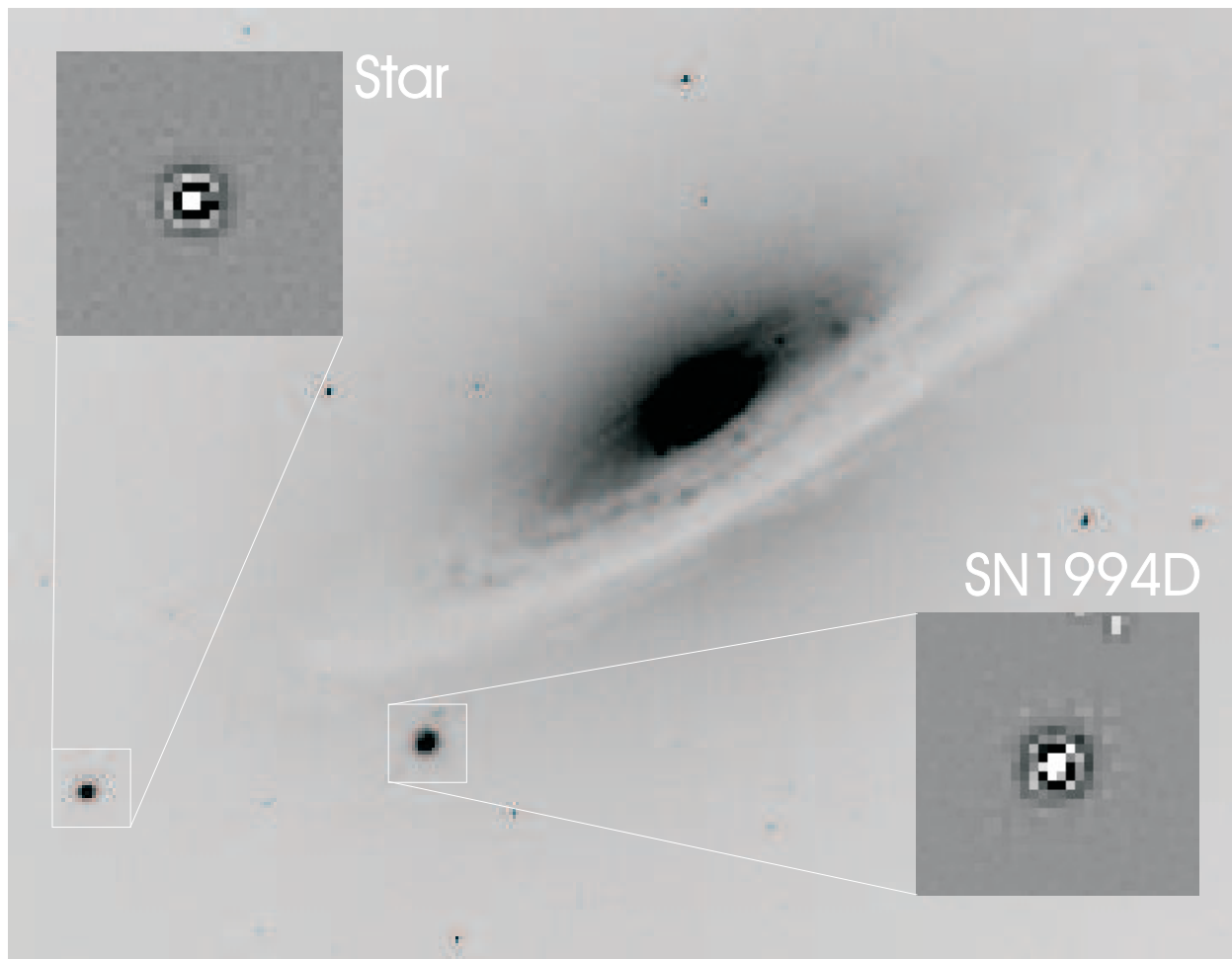


Fig. 2.— SN1994D as taken with HST. In the boxes a Galactic star and the SN are shown with the high spatial frequencies content of both original images enhanced in the same way by subtracting a smoothed version (with a 3×3 boxcar) of the same area. The Airy disks are clearly seen in both images, in spite of a small pixel size (equivalent to 0.046 arcsec) giving a poor sampling of the diffraction limit.

By comparing the HST–collected frame of SN1994D with that of a foreground Galactic star in the same field (see Fig.2) we see that both objects exhibit no deterioration of their Airy interference patterns. This constrains the Strehl deterioration parameter to $S > 0.2$ (else the first Airy rings will become invisible), and hence (by Eq. 6) places a lower limit for η at $\eta > 1.3$.

A separate investigation concerns the Hubble Deep Field high- z images. Spectroscopic follow-ups have shown that objects as distant as $z = 5.34$, corresponding to $L \approx 7.7\text{Gpc}$ are as small as 0.12arcsec (Spinrad et al., 1998). The distance adopted here is the comoving radial distance, as it is the summation over the journey of the photon of the length experienced by a comoving observer, where we assume the PSP exhibits in the same way. The exact value for L depends on the cosmological model used; here we have chosen $H_0 = 72\text{km/s/Mpc}$ and $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$ as given in Krauss (2002),

Since $\eta = 1.7$ is the ratio of this observed size to the diffraction limit capability of a $D = 2.4\text{m}$ telescope at the relevant wavelength of $\lambda = 814\text{nm}$, one can clinch PSP even further than before. We note in passing that, in reality, such a ratio for η understates the case against PSP, because while propagating from the source to us a photon initially had shorter wavelength, so that the quantity ΔL of Eq.(2), hence $\Delta\theta$ of Eq. (3), was larger in the past.

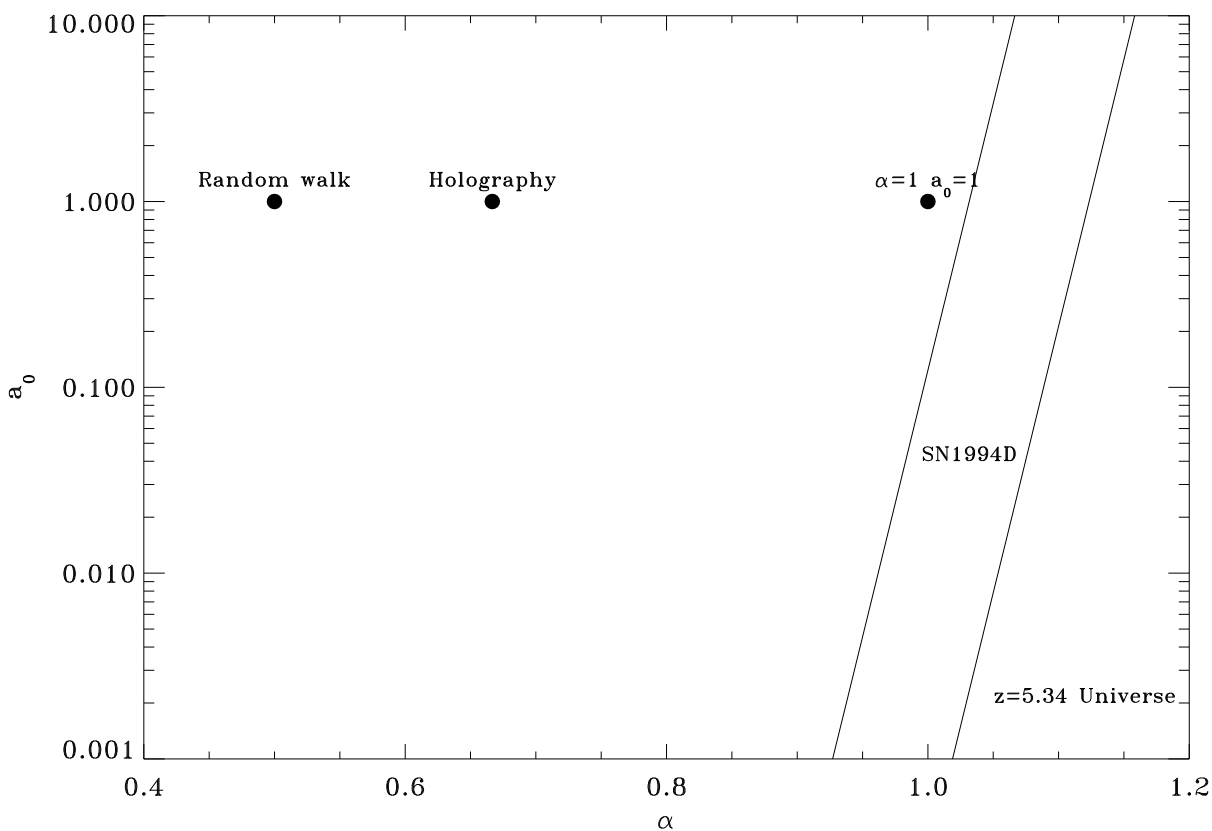


Fig. 3.— A portion of the parameter space $a_0 - \alpha$. The constraints imposed by the observations discussed in the text allow only the regions on the right side of the two oblique lines. Three points representing different PSP parameter choices are also shown, where we assumed the coupling coefficient $a_0 = 1$.

4. Discussion

In Fig. 3 implications of the two observations being analyzed thus far are plotted in a_0 - α space. We can see that a linear, first-order PSP characterized by $\alpha = 1, a_0 = 1$ is consistently excluded, as are the other cited phenomenologies with smaller α , for all reasonable values of a_0 . In particular, when $\alpha = 1$, the upper limit on the angular size of high- z objects requires that $a_0 < 3 \times 10^{-4}$.

The two benchmarks presented in the previous section were established with the most powerful instruments currently available (HST for measurement of the angular size and Keck for determination of the cosmological distance). It should be realized that, in general, the existence of PSP with $\alpha \sim a_0 \sim 1$ would render the universe unobservable at any appreciable redshift, due to the significant blurring of the images of point sources. This may be regarded as a form of Olber's paradox. In the case of the far universe, where observations require special technique, additional benchmarks could be envisaged.

Quantitatively, the limits given above for the exclusion of first order PSP understate the case, because (a) the errors were estimated conservatively - they would have assumed larger values had we propagated them at every step; (b) the wavelength of radiation from a distant source is shorter towards the source, meaning that our upper limit on a_0 should in reality be even smaller. Thus, in the same context as that of LH02, the possibility of $\alpha = 1$ may be ruled out with confidence.

Our conclusions may be compared and contrasted with other recent works, notably those of Jacobson, Liberati & Mattingly (2002) and Amelino-Camelia (2002). The former used X-ray observations of the Crab nebula to argue against PSP, its validity depends on the assumptions made about the physical processes in the Crab. The latter, however, proposed the *existence* of PSP effects as the reason why gamma-rays from a distant quasar survive their journey through the intergalactic medium to reach us. We note here also, that alternative interpretations are entirely possible.

In the framework of the assumptions made in LH02, PSP effects are excluded by the observations described in this Letter. Perhaps there exist some *ad hoc* explanations as to why first order PSP cannot be manifested as perturbation of a light pencil. As regards whether the present findings imply that the notion of structural space time at the Planck scales (a sort of *aether* embedded in the continuum where familiar physics holds) is untenable, or whether a subtle mechanism is at play to render such structures evasive, these questions are outside the scope of our Letter.

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